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DDD-0270

M. A. (Part - II) (Mathematics) (External) Examination

April/May – 2016

501 : Differential Geometry & Linear Algebra

Time : Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **M. A. (Part - II) (Mathematics) (External)**

Name of the Subject :

☛ **501 : Differential Geometry & Linear Algebra**

☛ Subject Code No. : 0 2 7 0 ☛ Section No. (1, 2,.....) : Nil

Seat No. :

Student's Signature

(2) All questions are compulsory.
(3) Notations used are standard.
(4) Figures at the right end of the first line of each question indicates full marks.

Q.1	(a)	Define: Null space and Range of a linear operator. Prove that the null space and range of a linear operator are subspaces of norm space v and w respectively.	[7]
	(b)	Let $T: p_2 \rightarrow R^3$ be defined by T(f)=X$(X_0, X_1, X_2)^T$ where $x_k = \int_0^{K+1} f(t)dt$;*K=0, 1, 2* then find *f(t)*.	[7]
	(c)	Define: Norm and Inner product space. Let $T: V \rightarrow W$ be a linear operator and dim(v)=n and dim(w) = m. If T is invertible then prove that m=n.	[6]
		OR	
Q.1	(a)	Let M=spam$\{v_1, v_2, v_3\}$, where $v_1 = (1,1,0,1)^T$, $v_2 = (3,1,2,-1)^T, v_3 = (1,2,2,0)^T$. Find the projection of f on M ; where f=$(0,2,1,1)^T$using normal and standard inner product.	[7]
	(b)	Suppose $\{A_k\}$; $k = 1 \dots n$ is a linearly independent set in F^m, and M=span$\{A_k\}$ $k = 1 \dots n$. S be the projection of f on M then prove that the normal equation are $A^H AC = A^H f$ and S=AC.	[7]
	(c)	If M = span$\{(1,1,1,1)^T, (1,2,2,-1)^T\}$. (i)Find an orthogonal basis for $M^\perp$. (ii) Find the projection of δ *on M and on* $M^\perp$.	[6]

Q.2	(a)	let A=$[A_1, A_2, \ldots A_n]$. Let $\langle , \rangle$ be the standard inner product on F^m then the following statement are equivalent. (i)S=Axis projection of y on M. (ii)Z=X minimizes $\|y - Az\|$ where z$\in F^n$.	[7]
	(b)	Find the orthogonal projection S of $\|t\|$ $on\ p_2$, using the real inner product $$\langle f, g\rangle = \int_{-1}^{1} f(t)g(t)dt.$$	[7]
	(c)	If M be a finite dimensional subspace of V and V is finite dimensional vector space. Then prove that (a)$V = M \oplus M^{\perp}$(b) $(M^{\perp})^{\perp} = M$.	[6]
		OR	
Q.2	(a)	If A be a $m \times n$ matrix and yϵF^m,prove that exactly one of the following is true. (i)Ax=y is constant. (ii)There exist Z such that $A^H z = 0\ and\ Z^H y \neq 0$.	[7]
	(b)	(1)M=span$\{A_1, A_2\}$ where $A_1 = (1,0,1,-1)^T$and $A_2 = (-1,0,2,1)^T$. Find a basis $M^{\perp}$. (2)Let V, W and U be a vector spaces, Show that for a finite dimensional vector space V, if $T: v \rightarrow w\ and\ s: w \rightarrow U$ are linear operators, then prove that r(ST)≤ $\min(r(S), r(T))$.	[7]
	(c)	State and prove Rank – nullity theorem.	[6]
Q.3	(a)	Derive equation of osculating plane. Find osculating plane at the point (0,0,0) on the helix : x=acosθ ; y=asinθ ; z=aθ.	[7]
	(b)	Define: principal Normal and Binormal. State and prove necessary and sufficient condition for the curve in space to be a plane curve.	[7]
	(c)	Define: Osculating Plane. Find equation of tangent line at any point on the circular helix X = acosu , Y = asinu , Z = cu.	[6]
		OR	
Q.3	(a)	State and prove Serret-Frenet formula.	[7]
	(b)	Define: Torsion. Derive its standard equation in 'DOT' notations.	[7]
	(c)	In usual notations prove that : $$x'''^2 + y'''^2 + z'''^2 = \frac{1}{(\rho\sigma)^2} + \frac{\rho'^2 + 1}{\rho^4}$$	[6]
Q.4	(a)	Define: Evolute. Derive formula to find Curvature and torsion of Evolute.	[7]
	(b)	Define: Osculating circle. Find the radius and centre of osculating circle.	[7]

	(c)	Find curvature and torsion for the curve X = a+acosu , y = asinu , Z = 2asin$\frac{u}{2}$.	**[6]**
		OR	
Q.4	**(a)**	Define: Envelop. The envelop of family of surfaces touches every members of the family of all points of its characteristics.	**[7]**
	(b)	Show that principal normal of one curve be Binormal of another curve if the relation a$(k^2+\tau^2)=bk$ must hold where a and b are constants.	**[7]**
	(c)	Find the envelop of the plane $3xt^2-3yt+z=\ t^3$; Where t is parameter.	**[6]**
Q.5	**(a)**	Computation of a projection using a spanning set.	**[7]**
	(b)	If $\{x\in R^4;x_1+x_2+x_4=0,x_1+x_3-x_4=0\}$ then find the matrix of the projection operator.	**[7]**
	(c)	If $v=T_1$ with ordered basis $\{1,\cos t,sint\}$. Define $T\colon v\to v$ by $T(x)=y$ if and only if $A[x]=[y]$.where$A=$ $\begin{bmatrix}2&-1&1\\3&0&1\\0&5&9\end{bmatrix}$ find $y=T(x)$ if $x=5+\sin t$.	**[6]**
		OR	
Q.5	**(a)**	If two parametric curves through any point of surface cut an angle w then prove that tan(w) = $\frac{H}{F}$.	**[7]**
	(b)	If θ is the angle between direction curves $pdu^2+2Qdudv+Rdv^2=0$ at any point (u,v) then prove that $\tan(\theta)=\frac{2H(Q^2-PR)^{\frac{1}{2}}}{ER-2FQ+GP}$	**[7]**
	(c)	Find the first magnitude at the right Idocoid X = ucosØ , Y = usinØ , Z = cØ.	**[6]**